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A CLIMATOLOGY OF EPSILON (ATMOSPHERIC DISSIPATION)

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ABSTRACT

Kolmogorov's structure functions for the longitudinal and transverse components of locally homogeneous isotropic turbulence are combined vectorially to obtain an expression which permits the evaluation of ϵ (atmospheric dissipation rate) from climatological data. This is used to derive climatological patterns of ϵ in the free atmosphere from Crutcher's upper wind statistics of the Northern Hemisphere. The latter are combined with Kung's boundary-layer values to estimate the distribution of total atmospheric dissipation over the Northern Hemisphere.

1. EPSILON AS A FUNCTION OF CLIMATOLOGICAL PARAMETERS

In a recent review of the methods of evaluating ϵ (the rate of kinetic energy dissipation in the atmosphere), it appeared that it could be determined from wind variability data. The theory for such an evaluation is provided by Kolmogorov's (1941*a*) second hypothesis of similarity of locally homogeneous isotropic turbulence. In such a field of turbulence and with the x axis along the mean vector wind, this gives for the wind components at points 1 and 2 a distance x apart

$$\sigma_u(x)^2 \equiv \overline{(u_2-u_1)^2} = 2(\sigma_u)^2[1-r_u(x)] = C(\epsilon x)^{2/3} \tag{1}$$

and

$$\sigma_v(x)^2 \equiv \overline{(v_2-v_1)^2} = 2(\sigma_v)^2[1-r_v(x)] = \tfrac{4}{3}\, C(\epsilon x)^{2/3},$$

i.e., the space variance or structure function (square of the Eulerian space variability, $\sigma_u(x)$, which in turn is a function of the standard deviation, σ_u, and the Eulerian space correlation, $r_u(x)$) is a function only of the separation distance and the intensity of the turbulence. The latter, by Kolmogorov's hypothesis, is determined by ϵ. Aside from the effect of orientation, equation (1) can be written by dimensional analysis. It can also be derived leading to theoretical as well as empirical values for the constant C. Values assigned C in the literature include $\tfrac{9}{5}$ (Kolmogorov, 1941*b*), $2^{2/3}$ (Obukhov and Iaglom, 1951), $\tfrac{4}{3}$ (MacCready, 1953), and approximately 2 (Hinze, 1959; Pond et al., 1963). In this study we use $C=2$ primarily because this yields the lowest estimates for ϵ.

The x-range of validity of (1) is presumed to be

$$l \ll x \ll L,$$
$$l^{-2} \equiv -\partial^2 r_u(x)/\partial x^2, \tag{2}$$

and

$$L \equiv \int_0^\infty r_u(x)dx.$$

In the atmosphere the microscale, l, ranges from a few millimeters near the surface to tens of meters near the jet stream and the macroscale, L, from a few meters to hundreds of kilometers (MacCready, 1953; Obukhov and Iaglom, 1959; Durst, 1954). By Taylor's (1938) so-called frozen turbulence approximation

$$x = \bar{u}t, \tag{3}$$

equation (1) becomes

$$\sigma_u(t)^2 = C(\epsilon \bar{u} t)^{2/3} \tag{4}$$

and

$$\sigma_v(t)^2 = \tfrac{4}{3}\, C(\epsilon \bar{u} t)^{2/3}.$$

Since meteorological wind observations are rarely resolved into components along and perpendicular to the mean vector wind, it is convenient to combine (4) vectorially to obtain

$$(\sigma_t)^2 = 7C(\epsilon \bar{u} t)^{2/3}/3. \tag{5}$$

It is worth noting that in data samples in which $\epsilon \bar{u}$ remains constant or can be replaced by its mean value equation (5) predicts

$$\sigma_t \propto t^{1/3}. \tag{6}$$

The vector time variability, σ_t, is related to the vector standard deviation, σ, and the vector Eulerian time lag correlation, r_t, by

$$(\sigma_t)^2 = 2\sigma^2(1-r_t), \tag{7}$$

which combines with (5) to yield

$$6\sigma^2(1-r_t)=7C(\epsilon\bar{u}t)^{2/3}. \quad (8)$$

The elusive quantity ϵ is now expressed entirely in terms of the climatological parameters σ, $\bar{u}$, r_t, and t.

2. VALIDITY ARGUMENTS

The validity of equation (8) appears to rest only on the validity of the following:

a) the Kolmogorov hypothesis (1941*a*) represented by equation (1),

b) the frozen turbulence hypothesis represented by (3), and

c) the time variability $t^{1/3}$ law represented by (6).

That is, if (1) and (3) are valid on any time scale and (6) is valid up to the lag period t at which σ_t (or r_t) is measured, then equation (8) would appear to hold despite the objections that might be raised concerning lack of homogeneous, isotropic, steady, and even three-dimensional turbulence on the scale represented by the lag period t.

The similarity theory of turbulence arising from Kolmogorov's hypothesis (1941a) can hardly be said to be proven, but a growing volume of consistent and confirming observed phenomena has led to an increasing number of useful applications of the theory in the fields of turbulence and diffusion. The Lagrangian form of (1) and Richardson's law (1926) of diffusion to which it leads have been derived by Lin (1960) without appealing to similarity.

The validity of the frozen turbulence approximation is obviously dependent on the ratio of the turbulent and mean flow energies, a quantity frequently expressed in terms of the gust factor

$$g\equiv\overline{[(u-\bar{u})^2]}^{1/2}/\bar{u}. \quad (9)$$

Ogura (1955) developed an equation for the structure function in a three-dimensional isotropic turbulent velocity field incorporating Kolmogorov's $-5/3$ power law and suggested a complete solution of the form

$$1-R(t)\propto t^m, \quad 2/3\leq \mathrm{m}\leq 1,$$

which reproduced the results obtained for the special cases studied—namely, $m=2/3$ for $g<<1$ and $m=1$ for $g>>1$. Gifford (1956) integrated Ogura's formula to obtain

$$1-R(t)=\frac{0.788gt}{1+0.891g}+0.614\left(\frac{t}{1+0.891g}\right)^{2/3}$$

for $t<<T_0$ where T_0 is the period of the eddies of maximum energy. This not only reproduces the special cases of Ogura but when plotted as a function of g reveals that for the usual atmospheric range of $g\leq 1$ the time-correlation curve is very close to the limiting case for $g=0$ which is consistent with (4). Gifford concluded that the conversion of Eulerian space to Eulerian time correlation through equation (3) could be expected to apply closely in the atmosphere as well as in the wind tunnel, a judgment with which Taylor (1957) concurred following a reexamination of empirical data.

In spite of Gifford's findings (1956), we chose to improve the frozen turbulence approximation by replacing $\bar{u}$ in equation (8) by $\bar{s}$, the mean scalar wind, leading to

$$\epsilon=[3\sigma^2(1-r_t)/7]^{3/2}/\bar{s}t. \quad (10)$$

This reduces the values of ϵ from (8) by a factor equal to the steadiness of the wind and, of greater importance, eliminates the singularities that would have occurred whenever the mean vector wind approached zero.

As empirical evidence for both (3) and (1), MacCready (1953), Taylor (1955, 1961), and Ball (1961) found atmospheric observations to be consistent with the component forms of equation (8) for equivalent distances generally exceeding the height of the observation several fold. Their observations were taken with special small-sample fast-response equipment at elevations of 7.5 cm to 153 m. Accepting their reports as validating equation (8) for evaluating ϵ over equivalent distances ($\bar{u}t$) comparable to the elevation of the observation point, its validity for greater lag times (equivalent distances) depends only on the validity of (6). In a companion study, published wind variability data were found to be generally consistent with (6) for lag periods up to 6 hr (Ellsaesser, 1969).

3. DATA AND RESULTS

Encouraged by this evaluation we proceeded with the proof-of-the-pudding by preparing climatological maps of ϵ. For this purpose we choose to use a lag period of 6 hr since this is both the shortest interval at which a significant amount of data on r_t are available and is the longest period for which (6) is generally valid. Values of vector standard deviation (σ) and mean scalar winds ($\bar{s}$) were interpolated visually at 10° intersections from Crutcher's (1959–1962) maps and where necessary were extrapolated to 10° N. on nothing more than synoptic experience. To keep the workload within bounds, only spring values were read, since previous investigations have shown these to be most representative of annual means. The value r_t=0.793 for t=6 hr was taken from Ellsaesser (1960). This is an average of 500-mb values for spring at 21 U.S. Air Force stations in North America. This value was used for all latitudes and altitudes in the absence of better estimates of r_6. Durst (1954) and Charles (1959) did not indicate significant variation of r_t with altitude.

The results for the 50-, 100-, 200-, 300-, 500-, 700-, and 850-mb levels are shown in figures 1–7. Please bear in mind that these are machine produced. The utility analysis routine connects by straight lines linearly interpolated points on the boundary of each 10 by 10° cell. Values of ϵ

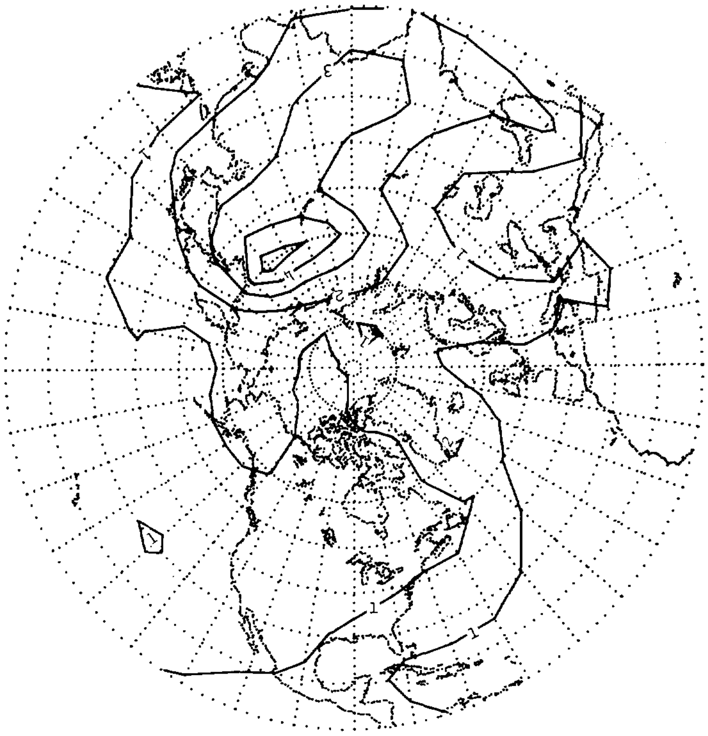

FIGURE 1.—Contour analysis of ϵ (dissipation) computed from equation (10) and Crutcher's (1959–1962) maps for 50 mb. Units are cm²/sec³ (erg gm⁻¹ sec⁻¹). Maximum over Siberia is believed to be data weaknesses in Crutcher's data.

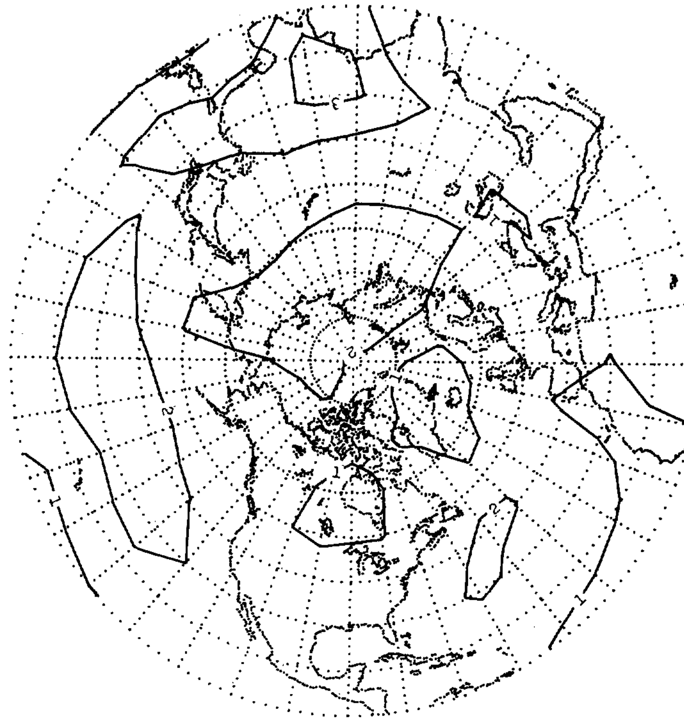

FIGURE 2.—Same as figure 1 except for 100 mb.

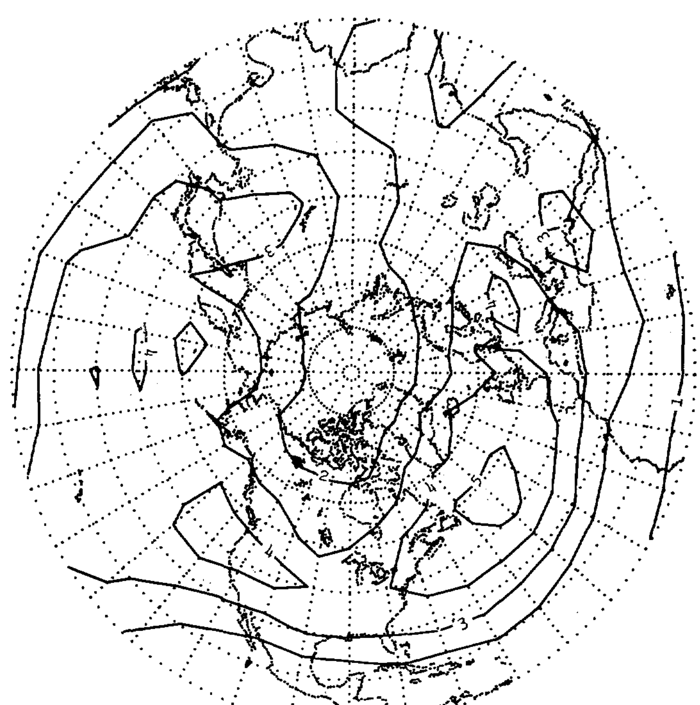

FIGURE 3.—Same as figure 1 except for 200 mb.

were computed in the unit cm^2/sec^3 or erg gm^{-1} sec^{-1} and were assumed equivalent to 10^{-3} watts m^{-2} mb^{-1} (ignoring a 2-percent correction). Thus numbers for point- and pressure-averaged values of dissipation, ϵ and $\bar{\epsilon}$, in cm^2/sec^3 can be compared directly with E, the pressure integral of ϵ, when integrated through 1000 mb and expressed in watts/m².

Figure 8 displays E_f, the pressure integral of ϵ from 0 to 900 mb, which is presumed to represent a 5-yr mean of the dissipation in the free atmosphere in spring (approximates annual mean) in watts/m². We see a minimum of less than 1 watt/m² at the 10° N. boundary increasing to a jet stream maximum exceeding 2 watts/m² at most longitudes and then dropping to a relative minimum between 1 and 2 watts/m² at the Pole. We also see a continental minimum and an oceanic maximum, the former most pronounced over the largest continent but the latter most pronounced over the Atlantic (possibly because of the greater density of observational data).

Table 1 shows area weighted means of figures 1–8 for the seven geographical regions: Arctic, 70° and 80° N.; Tropics, 10° and 20° N.; the four longitudinal sectors 0°–130° E., 140°–230° E., 240°–290° E., and 300°–350° E. extending from 30° to 60° N.; and the total hemisphere.

It should be noted that at each point at which arbitrariness appears in our evaluation of ϵ, we chose that course which would lead to the smaller values. In applying the theory to meteorological balloon wind soundings, an observation system designed to eliminate much of the subsynoptic scale turbulence from the data, we expect a further reduction in the computed values of ϵ. For these reasons the values in figures 1–7 and table 1 are considered to represent lower limits for ϵ and E_f. Values for E_f reported by Smith (1955), Jensen (1961), Holopainen (1963), and Kung (1966*a*, 1966*b*, 1967), ignoring the 12 GMT

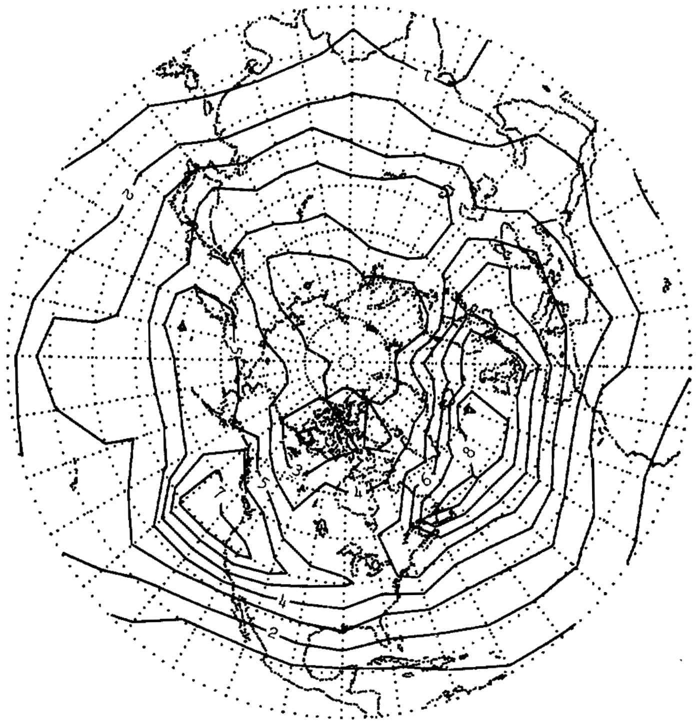

FIGURE 4.—Same as figure 1 except for 300 mb.

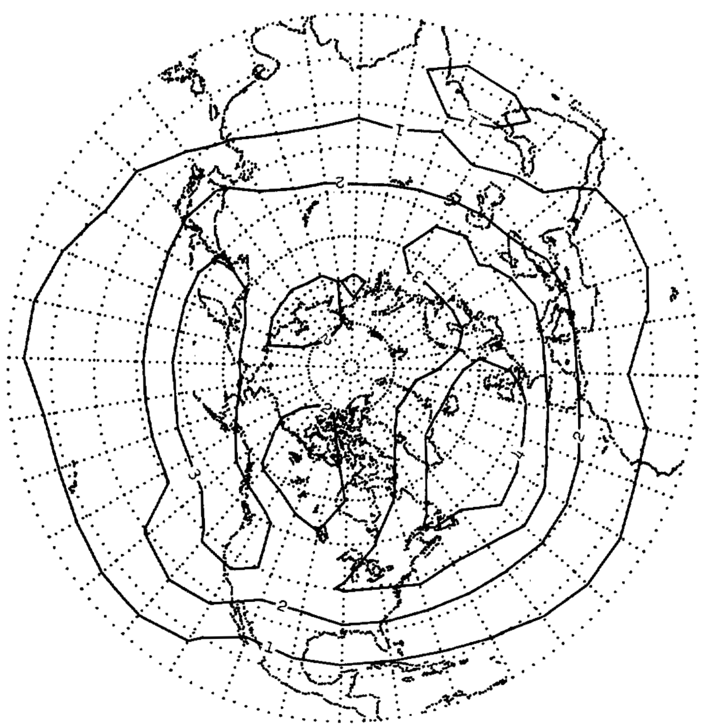

FIGURE 5.—Same as figure 1 except for 500 mb.

results of the latter for reasons discussed later, range from 4 to 6 watts/m^2 or about a factor of 2 larger than our results. This factor matches our crude estimate of the enhancement factor anticipated if we had turbulence-type observations from which to evaluate ϵ from (1).

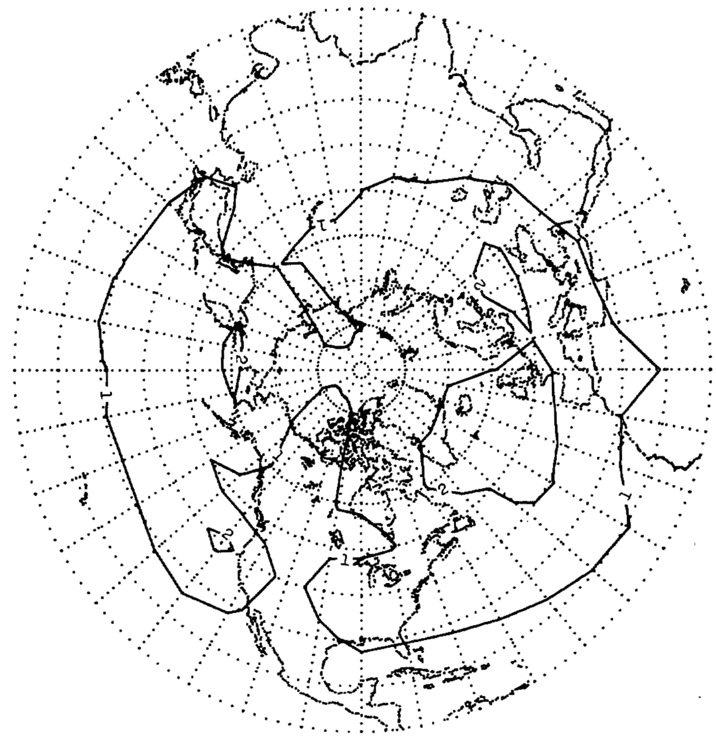

FIGURE 6.—Same as figure 1 except for 700 mb.

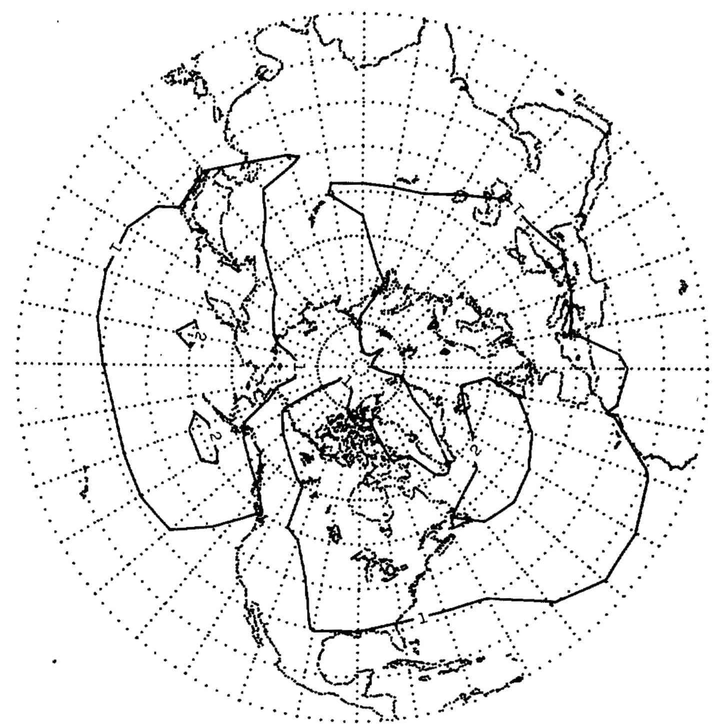

FIGURE 7.—Same as figure 1 except for 850 mb.

4. MISCONCEPTIONS CONCERNING ATMOSPHERIC DISSIPATION

The oceanic maxima of ϵ and E_f appear anamolous in view of the two prevailing misconceptions that the bulk of atmospheric dissipation occurs in the surface boundary layer and that the latter should vary directly with surface roughness. The evidence against these misconceptions is not yet conclusive but it has begun to emerge.

Using the kinetic energy equation Holopainen (1963) found that 6.2 watts/m² or 60 percent of the dissipation over the British Isles for January 1954 occurred above 900 mb. Applying the same method to North American data, Kung (1966*a*, 1966*b*, 1967) found for the first 6 mo of

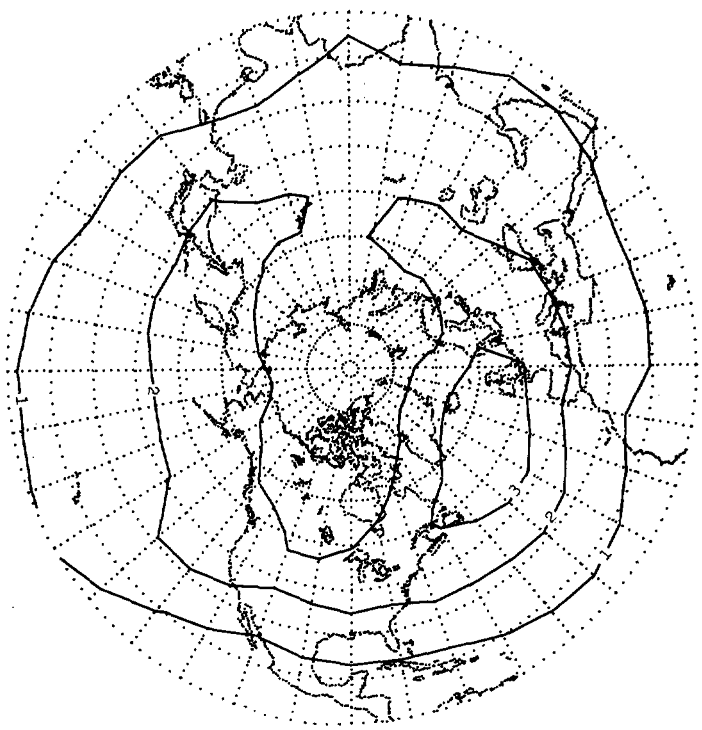

FIGURE 8.—Contour analysis of E_f (total dissipation in the free atmosphere) computed as a 0- to 900-mb pressure integral of figures 1–7. Units are watts/m².

00 GMT data that 4.51 watts/m² or 71 percent occurred above 900 mb (925 in winter); for the first 11 mo of 00 GMT data, 4.91 watts/m² or 69 percent occurred above 875 mb; and for 5 yr of 00 and 12 GMT data, 2.05 watts/m² or 49.8 percent occurred above the lowest 100-mb layer (i.e., above 868 mb). Considered more significant is Kung's (1966*a*) finding of a near-perfect balance between kinetic energy generation and dissipation in the boundary layer and his confirmation of findings by Jensen (1961), Holopainen (1963), and Smagorinsky et al. (1965) that the vertical transport of kinetic energy at or near the top of the boundary layer is very small in comparison with the generation in the boundary layer. He interpreted these findings to indicate that most of the *kinetic energy generated in the free atmosphere is dissipated in the free atmosphere.*

The fraction of the kinetic energy generated throughout the depth of the atmosphere which is released $(-\mathbf{V}\cdot\nabla\phi)$ above the boundary layer has been calculated to be 61 percent above 811 mb in a nine-level general circulation model (Smagorinsky et al., 1965) and 67 percent above 850 mb from 5 yr of North American data (Kung, 1967, table 3). While Smith (1955) and Jensen (1961) did not supply this estimate explicitly, it is apparent from their illustrations that the bulk of the release occurred above 800 mb. Only Holopainen (1963) indicated that the bulk of the release occurred in the boundary layer (below 900 mb); yet as reported above, he found that 60 percent of the dissipation in the surface to the 200-mb layer occurred above 900 mb. The balance is accounted for by transports and local change.

As for the oceanic maximum, Kung (1966*a*) pointed out that in Lettau's (1962) dissipation equation based on surface roughness and geostrophic wind the latter enters to the third power, and since it is generally larger over the oceans, it can well compensate for the smaller

TABLE 1.—*Area weighted mean dissipation in the atmosphere as a function of pressure and geographic region (ergs gm⁻¹ sec⁻¹)*

Pressure level (mb)	Geographical region						
	North Atlantic (300°–350° E.)	North Pacific (140°–230° E.)	North America (240°–290° E.)	Eurasia (0°–130° E.)	Northern Hemisphere	Arctic (70° & 80° N.)	Tropics (10° & 20° N.)
50	0.939	1.158	0.812	2.008	1.251	1.311	1.029
100	1.616	1.836	1.228	1.524	1.480	1.201	1.416
200	4.066	3.439	3.300	2.666	2.587	1.958	1.843
300	5.285	4.284	4.296	3.617	3.014	3.082	1.296
500	3.233	2.399	2.416	1.965	1.768	2.274	0.763
700	1.901	1.279	1.095	1.128	0.991	1.293	0.474
850	1.629	1.325	1.142	0.938	0.878	1.050	0.375
	Layer integrals (watts/m²)						
Free atmosphere (0–900 mb)	2.574	2.080	1.945	1.782	1.553	1.659	0.844
From Kung (1963) (900–1000 mb)	1.510	1.645	2.385	0.881	1.113	1.114	0.630
Total	4.084	3.725	4.330	2.663	2.666	2.773	1.474

surface roughness. Also, in his North American study, Kung (1966*a*, 1966*b*, 1967) found a consistent export of kinetic energy from North America to the Atlantic. For the 5 yr of data, of the kinetic energy generated over North America 52 percent was exported to the Atlantic in winter, 34 percent in summer, and 46 percent on an annual basis. Unless one postulates a marked reduction in kinetic energy generation over the Atlantic or a convergence of kinetic energy farther downstream, this requires a greater dissipation over the Atlantic than over North America.

5. DISSIPATION IN BOUNDARY LAYER

To complete our climatology requires an evaluation of ϵ in the surface boundary layer. Such an evaluation from equation (10) applied to standard meteorological observations is not possible since the distance from the surface restricts the scale size within which homogeneous isotropic turbulence may be approximated. The averaging scale of standard winds-aloft observations is of the order of a volume a half a kilometer deep and several kilometers long. The scales of turbulence permitted near the surface are obviously severely damped by such averaging.

Fortunately, an estimate of E_b, the dissipation in the boundary, was available. Using Lettau's theory (1962), his own estimates of the surface roughness, and 1000-mb geostrophic winds for the period 1945–1955, Kung (1963) computed E_b over a network of 360 points spanning the Northern Hemisphere between 25° and 70° N. Figure 9 is our attempt to portray the geographical variations of his results for spring. To obtain this, his mean value for 25° N. was assigned to 10° and 20° N. and his 70° N. value to 80° and 90° N. Other points were determined by linear interpolation. Table 1 shows the area means of figure 9. Note that our geographical subdivision differs from Kung's (1963) and accounts for the lack of agreement between the values of E_b in table 1 and those in his report. The total dissipation, $E=E_f+E_b$, appears in the last line of table 1.

Except over North America, Kung (1963) computed only meridional profiles by geographical regions. This means that in the latter regions the cube of the average geostrophic wind is compared with an average of the cube of the geostrophic wind over North America. Kung estimated that this procedure could lead to underestimates in ϵ of 30 percent or more. An adjustment of this magnitude is insufficient to extend the continental minimum for ϵ, evident over Eurasia, to North America. However, the above was not the only nonlinear effect involved. Kung also assumed a constant surface roughness for the oceans, although suspecting a direct variation with wind speed, at least for relative speeds above some critical value. He used monthly averages of the geostrophic wind, and his meridional profiles reveal significant minima at 30° and 40° N. compared to 25°, 35°, and 45° N. Both of these factors further diminish our values of E_b including those

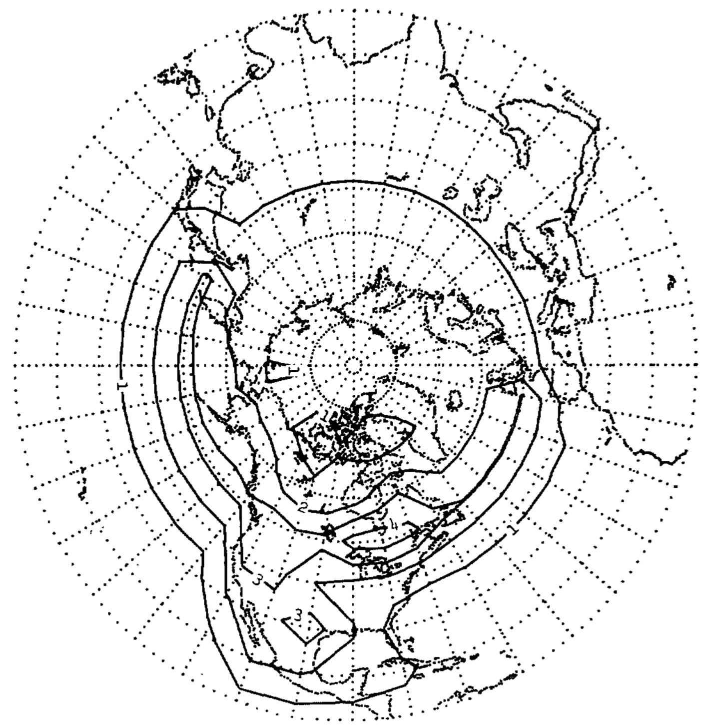

FIGURE 9.—Contour analysis of E_b (total dissipation in the boundary layer from Kung, 1963). Units are watts/m².

for North America. We believe these factors justify acceptance of Kung's North American value as most representative and enhancement of the others sufficiently to yield at least comparable values over the oceans, i.e., by about 50 percent.

7. THE VARIATIONS OF EPSILON

After multiplying the data by the adjustment factors indicated above (2 for our data and 1.5 for Kung's, 1963, data outside North America), we arrive at the adjusted values for atmospheric dissipation shown in figure 10 and table 2. Figure 11 shows the adjusted bar diagram pressure profiles of ϵ. Kung's (1963) boundary values were included by assuming that his E_b applied to the 1000- to 900-mb layer. This treatment gives a distorted picture of the logarithmic variation of ϵ near the surface, where several investigators have estimated values of ϵ exceeding 1000 cm ^{2}/sec ^{3} (MacCready, 1953; Ball, 1961; Lettau, 1954; Taylor, 1952). The profiles of ϵ in figure 11 agree quite well with those of Kung (1966*a*, 1966*b*, 1967) and Smagorinsky et al. (1965) and the generation ($-\mathbf{V}\cdot\nabla\phi$) profiles of these, Smith (1955) and Jensen (1961). Note, however, that they differ markedly in the free atmosphere from the completely logarithmic profiles postulated by investigators in the fields of turbulence and diffusion (Ball, 1961; Wilkins, 1963).

An analysis of the seasonal variations of dissipation by Kung (1963) and energy conversion by Wiin-Nielsen and Drake (1966) and Krueger et al. (1965) suggests that both spring and fall values of atmospheric dissipation are near the annual mean while winter values are 50 percent greater and summer values 50 percent smaller than the annual mean. Kung's (1967) study separating 00 and 12 GMT data yields a confusing picture of the seasonal variation of dissipation. In his figure 11, the 00 GMT data indicate a spring (April) seasonal (monthly) maximum and a winter (December) minimum while the 12 GMT data indicate a winter (December) maximum and a summer (July) minimum. More difficult to accept is the 12-fold decrease in total dissipation in summer between 00 GMT and 12 GMT indicated by his table 4. His figure 2 reveals that most of this decrease is due to large negative values of 12 GMT dissipation above 300 mb, which are sufficient to yield negative values of E for June and July. Only a modest amount of the decrease occurs in the lower troposphere, where diurnal variation in surface heating and convection could provide a logical explanation. These unexplained diurnal and seasonal variations make the interpretation of Kung's (1967) results questionable. We are inclined to disregard his 12 GMT data above 300 mb and his 00 GMT data for December but can offer no rationale that would tend to increase confidence in the rest of his results.

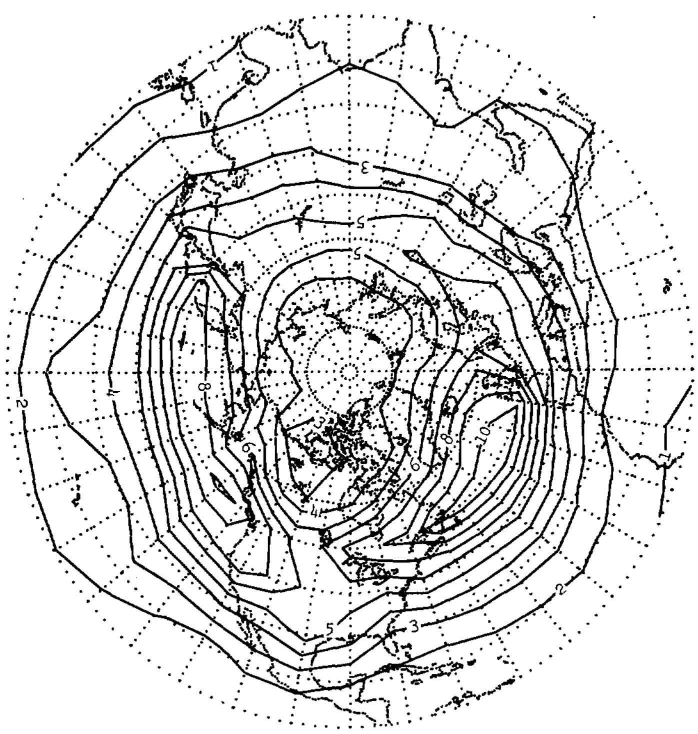

FIGURE 10.—Contours of total adjusted (E_f multiplied by 2, E_b by 1.5, except over North America) atmospheric dissipation. Units are watts/m².

Kung (1963) reported standard deviations of monthly area mean values of dissipation of 30 to in excess of 50 percent of the seasonal mean values, and Kung's (1966*a*, 1966*b*, 1967) illustrations suggest equally large day-to-day variations.

8. DISCUSSION

Despite the agreement with previous studies, our climatology of ϵ must be regarded as experimental. It represents an extension of Kolmogorov's hypothesis to time scales over which the assumptions of steady homogeneous and isotropic three-dimensional turbulence are clearly not satisfied. However, this has also occurred in other applications of his hypothesis. Further, in so far as the evaluation of ϵ is concerned, this does not appear to be a serious objection so long as time variability of the horizontal wind varies as the cube root of the lag period as predicted by equation (6). Aside from theoretical reservations, we can be confident that the values in table 1 and figures 1 through 8 represent lower limits for ϵ. In view of the deliberate reduction in small-scale short-period variance of meteorologically observed winds aloft, an adjustment upward by a factor of 2 (comparable to a reduction of true variance not exceeding 37 percent) to

TABLE 2.—*Area weighted mean adjusted (free atmospheric values increased by 2, boundary values by 1.5 except over North America) dissipation in the atmosphere as a function of pressure and geographic region (ergs gm⁻¹ sec⁻¹)*

Pressure level (mb)	Geographical region						
	North Atlantic (300°–350° E.)	North Pacific (140°–230° E.)	North America (240°–290° E.)	Eurasia (0°–130° E.)	Northern Hemisphere	Arctic (70° & 80° N.)	Tropics (10° & 20° N.)
50	1.878	2.317	1.623	4.015	2.503	2.622	2.058
100	3.233	3.672	2.455	3.048	2.960	2.402	2.831
200	8.133	6.878	6.600	5.332	5.174	3.915	3.686
300	10.569	8.567	8.591	7.235	6.029	6.165	2.592
500	6.465	4.798	4.832	3.930	3.537	4.548	1.525
700	3.803	2.558	2.190	2.256	1.982	2.586	0.947
850	3.259	2.649	2.284	1.876	1.756	2.099	0.750
			Layer integrals (watts/m²)				
Free atmosphere (0–900 mb)	5.148	4.161	3.890	3.564	3.105	3.318	1.689
From Kung, 1963 (900–1000 mb)	2.229	2.399	2.440	1.322	1.529	1.578	0.897
Total	7.377	6.559	6.329	4.885	4.634	4.896	2.586

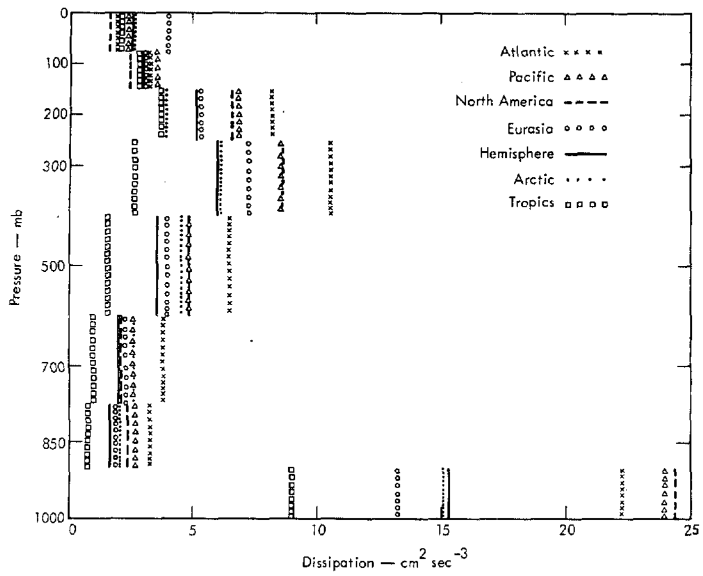

FIGURE 11.—Pressure profiles of adjusted (free atmospheric values multiplied by 2, boundary values by 1.5, except over North America) values of ϵ integrated to obtain values in table 2. Boundary-layer value obtained by assuming E_b from Kung (1963) applies to 1000- to 900-mb layer.

bring them into agreement with published estimates does not seem unreasonable.

Other than the smoothing introduced by winds-aloft observations, the weakest link in the present evaluation is considered to be the calculation of σ_t from σ by a constant $r_6=0.793$. Not many estimates of this parameter are available for lag periods of 6 hr or less. Durst (1954) arrived at a value of $r_6=0.88$ for all altitudes and seasons based almost exclusively on Larkhill data. This value would reduce the computed values of ϵ by a factor of 2. Since not one of the 21 stations given in Ellsaesser (1960) had such a large value of r_6, their spring mean was adopted as a more representative value. This may account for the Atlantic maximum of ϵ since use of a value closer to Durst's (1954) value of r_6 in this region would have resulted in smaller values of ϵ. Despite this, we still cling to the belief that oceanic maxima in ϵ are real.

In view of the critical importance which ϵ is assuming in applications of turbulence and diffusion theories and the tenuity of the estimates of ϵ frequently used, it is hoped that this climatology of ϵ, albeit experimental, will prove useful. Meanwhile we hope both to increase our confidence in equation (10) and to improve our estimates of ϵ by using observed values of σ_t for $t \leq 6$ hr. In particular, we hope this approach will shed some light on the question raised by Kung (1967) concerning the diurnal variation of ϵ.

ACKNOWLEDGMENT

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